DARIEN PUBLIC SCHOOLS

CURRICULUM GUIDE

Precalculus

DARIEN PUBLIC SCHOOLS

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DATES

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SECTION I – COURSE INFORMATION

STATEMENT OF PHILOSOPHY

DARIEN HIGH SCHOOL MATHEMATICS DEPARTMENT PHILOSOPHY

We believe in creating learning environments where students practice and acquire the knowledge of mathematics. We believe that students should be able to proficiently apply a range of numerical, algebraic, geometric and statistical concepts and the skills to formulate, analyze and solve real world problems. The learning environments will facilitate inquiry, use of technology and the exploration of real world phenomena. It will support continuous development of mathematical skills and the appreciation of mathematics as a discipline. Our mathematics program seeks to graduate students who will possess a sense of numbers, data analysis, spatial relationships, symbolic representation and the ability to communicate mathematics with others.

DISTRICT MATHEMATICS PHILOSOPHY

Mathematics is a vigorous and growing discipline – a universal language useful for communication and research in other disciplines. We want our students to reason and communicate mathematically, to be mathematical problem-solvers, to value mathematics and to feel confident in their ability to use mathematics. Creating such a foundation necessitates a well-articulated and developmentally appropriate mathematics program for all, developing the mathematical power of each.

Mathematics is more than a collection of concepts and skills to be mastered. It is the exploration of ideas and concepts, the understanding of relationships, the ability to make predictions, to analyze data, to estimate results, to communicate ideas and to solve problems in this ever-changing world. It is no longer limited to the study of complex calculations and formulas. We are moving from a curriculum often dominated by memorization of isolated facts and procedures to one that emphasizes conceptual understandings, multiple representations, deliberate connections and mathematical problem solving. Rather than being a transmitter of knowledge, the teacher becomes a facilitator of learning, guiding, questioning, listening, clarifying and creating an environment in which the student is an active participant in learning.

The needs of today's society demand that all students become mathematically literate to function effectively. It will be necessary for our students to be able to use mathematics in their personal lives, further studies and future workplaces. As educators, we must recognize that students have differing abilities, performance levels, needs and interests

and provide them with the best mathematics education possible so that they may achieve their personal ambitions and career goals.

Too often, students have learned to compute without understanding why the computation procedures make sense or how they apply to their lives. Instruction must focus on the behaviors that contribute to the development of mathematical thinking and number sense – explaining procedures used, justifying reasoning, judging the reasonableness of solutions and reflecting on the application of concepts.

When students gain knowledge from meaningful experiences, they are much more likely to retain and use what they have learned. Sound practice in the teaching of mathematics means that students are guided to use concrete materials and explore ideas with classmates. In this way, knowledge evolves from personal experience.

The fundamental objective of education has always been to prepare students to be contributing members of the society in which they live. The objectives of this mathematics curriculum support and affirm this tradition.

PROGRAM GOALS

- Introduce students to the language of trigonometry and advanced algebra.
- Make students more comfortable with trigonometric and algebraic graphs.
- Have students learn to apply the six basic trigonometric functions.
- Enable students to use algebra and/or trigonometry to define and solve problems.
- Provide the opportunity for students to develop mathematical and logical reasoning.
- Show how pre-calculus is connected with calculus, other disciplines and real world situations.
- Make students aware of the historical development of advanced algebra and trigonometry.
- Allow students to utilize the graphing calculator to make and analyze graphs.
- Introduce students to the concept of limits and differential calculus.

OVERVIEW

Precalculus is a two-semester course. The first semester is Trigonometry and includes the Law of Sines and Law of Cosines, circular functions and trigonometric equations. Through the use of real life examples, students' understanding of sinusoidal functions is developed and strengthened. The second semester covers topics in Advanced Algebra. This includes the study of rational, polynomial, exponential and logarithmic functions, conics and an introduction to differential calculus. The TI-83 or TI-83 Plus graphing calculator is required and used extensively in the course.

This class is ideal for a college bound student who may be interested in pursuing a degree in engineering, business or mathematics. The material covered will provide the student with a solid foundation in advanced algebra. The math background needed to successfully complete the course is a solid understanding of Geometry and Algebra 1 and Algebra 2. The class has both a 300 level and 400 level option open to seniors and juniors.

ESSENTIAL QUESTIONS

1. How can we better understand the world around us by analyzing and recognizing trigonometric functions?
2. How can we develop ways to explore and analyze higher-level mathematics in order to assist us in improving our problem solving skills?
3. How can we use mathematical modeling to enable us to more precisely explain, organize and understand the world around us?
4. How can we communicate our mathematical thinking clearly?
5. How can we organize and consolidate our knowledge and sense of numbers and mathematics?

PROCESS SKILLS

- Reading (comprehending)
- Reading (analyzing)
- Reading (appreciating)
- Writing mathematical equations
- Speaking the language of algebra and trigonometry
- Listening
- Viewing
- Studying
- Reasoning and reflecting
- Using learning resources, manipulatives, technology
- Working independently and collaboratively
- Designing
- Creating
- Quantifying
- Understanding mathematical operations
- Computing
- Problem solving
- Graphing
- Using technology
- Applying trigonometry
- Applying algebra
- Others

STUDENT PERFORMANCE SUMMARY

- Cooperative learning
- Student board demonstration of math skills and problem solving
- Class discussions on selected topics
- Mathematical modeling with written explanation
- Proficiency in using TI-83 calculator to solve problems
- Explaining to other students how to use TI-83 calculator
- Interpreting problems and using TI-83 graphing technology to solve problems.
- Tests and quizzes
- Homework
- Class participation

GRADING GUIDELINES

	Expectations of students	**% of Grade**
Homework	100% of all assignments	0 – 20%
Notebook	all notes maintained	
Quizzes	all quizzes taken	30 – 40%
Tests	all tests taken	45 – 60%
Mid year exam		20% of semester grade
Final exam		20% of semester grade
Projects	completed on time	0 – 20%
Class participation	every day	0 – 5%

SECTION II - Units of Study

SUMMARY OF UNITS

Unit Title	Duration (weeks)
ONE: Unit Circle and Trig Functions; Circular Functions	**4-5**
TWO: Triangles and Trigonometry	**4-5**
THREE: Sinusoidal Functions	**3**
FOUR: Identities	**2**
FIVE: Trigonometric Equations	**2**
***SIX: Parametric Equations**	**1-2**
SEVEN: Relations; Functions; Inverses; Systems; Graphs	**4-5**
EIGHT: Polynomial Functions and Rational Functions	**2-3**
NINE: Exponential Functions and Logarithmic Functions	**2-3**
TEN: Arithmetic , Geometric, and Infinite Sequences and Series	**1-2**
ELEVEN: Conics	**2-3**
TWELVE: Limits, Derivatives, and Curve Sketching	**2-3**
THIRTEEN: Algebraic Methods of Data Analysis	**1-2**

UNIT 1 : Circular Trig Functions; Unit Circle

ESSENTIAL QUESTIONS

1. Can the student measure angles in radians and degrees?
2. Can the student determine angles coterminal with a given measure?
3. Can the student determine the reference angle for a given angle?
4. Can the student determine the arc length and sector angle, given the central angle measure?
5. Can the student write the appropriate trigonometric function for an angle of rotation?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Geometry from a Synthetic Perspective; Geometry from an Algebraic Perspective; Trigonometry

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Trig functions – sine, cosine, tangent, cotangent, secant, cosecant
2. Trig function values for special measures – multiples of 30° and 45°
3. Trig function values for negative measures and values greater than 2 radians
4. Unit circle
5. Coterminal angles and reference angles
6. Angle measurement in degrees and/or radians
7. Arc length and area of a sector

Skills:

1. Convert from radians to degrees and vice versa
2. Use the unit circle to locate all positions that are multiples of 30° and 45°
3. Sketch an angle in standard position
4. Determine the values of the six trig functions of an angle in standard position, given a point on the terminal side of the angle of rotation
5. Express the six trig functions in terms of ***x, y,*** and ***r***
6. * Determine apparent size
7. Use knowledge from the unit circle to determine whether a trig function is positive, negative, or zero (noncalculator)

8. Use knowledge of 30° and 45° and their multiples to state exact trig function values (noncalculator)
9. Use knowledge of reference angles to determine trig function values for measures < zero radians and > two pi radians

*****Covered in the 400-level course

VOCABULARY

Angle of rotation; arc length; *apparent size; cosine; coterminal angles; circular function; initial side; minutes; quadrantal angle; radians; reference angle; revolution; rotation; second; sector; sine; standard position; terminal side; tangent; unit circle;

ACTIVITIES

Determine angle measurements.
Use the Unit circle

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 5
400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 7

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 2 : Triangles and Trigonometry

ESSENTIAL QUESTIONS

1. Can the student use **SOH-CAH-TOA** to solve right triangles?
2. Can the student solve a triangle using Law of Sines?
3. Can the student determine whether a triangle has zero, one or two solutions?
4. Can the student solve triangles using Law of Cosines?
5. Can the student interpret a real world situation and solve using trigonometry?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Geometry from a Synthetic Perspective; Geometry from an Algebraic Perspective; Trigonometry

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. SOH-CAH-TOA mnemonic for trig functions in a right triangle
2. Law of Sines
3. Law of Cosines
4. Area of a triangle or polygon

Skills:

1. Use SOH-CAH-TOA to solve for an angle measure or side length in a right triangle
2. Use Law of Sines to solve a triangle
3. Use Law of Cosines to solve a triangle
4. Determine the number of triangles possible from Side-Side-Angle; solve for the related triangle(s)
5. Use Law of Sines or Law of Cosines to determine the area of a polygon
6. Read and interpret real world situations and solve.

*****Covered in the 400-level course

VOCABULARY

Ambiguous case (SSA); apothem; angle of elevation; angle of depression; altitude; adjacent leg; opposite leg; hypotenuse; segment of a circle; sector

ACTIVITIES

Drawing and solutioning any type of triangle.

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Land surveying
Nautical navigation

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 5
400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 9

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 3 : Sinusoidal Functions

ESSENTIAL QUESTIONS

1. Can the student use sinusoidal functions to set up an equation and solve related questions?
2. Can the student use sinusoidal functions to model real world periodic behavior?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Geometry from an Algebraic Perspective; Trigonometry

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Graphs of sinusoidal functions
2. Equations of sinusoidal functions
3. *Angular and linear velocity

Skills:

1. Given the amplitude, period, phase shift, and vertical shift, write a related equation
2. State the amplitude, period, phase shift, and vertical shift of a sinusoidal function
3. Graph several periods of a sinusoidal function
4. Determine an equation from the graph of a sinusoidal function
5. Set an appropriate calculator Window and use the graphing calculator to graph the function and answer related questions
6. Read and interpret real world situations and solve using trigonometry

*****Covered in the 400-level course

VOCABULARY

Amplitude; *angular velocity; axis of rotation; horizontal shift; *linear velocity; period; phase shift; vertical shift

ACTIVITIES

Evaluating motion of tides, Ferris wheels, space ships, temperature etc.

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 6

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 8

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 4 : Identities

ESSENTIAL QUESTIONS

1. Can the student use algebraic techniques to simplify trigonometric expressions?
2. Can the student use trig identities to simplify other trigonometric identities?

NCTM STANDARDS

Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Geometry from an Algebraic Perspective; Trigonometry

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Verifying identities
2. Double-angle identities
3. Reciprocal identities
4. Pythagorean identities
5. Quotient identities
6. Symmetry identities
7. *Co-function identities
8. *Sum and difference formulas for sine, cosine, and tangent
9. Simplifying expressions using identities for substitution

Skills:

1. Use algebraic techniques (factoring, simplifying expressions, etc.) to verify an identity
2. Use trig substitution, derived from identity information, to verify a given identity

***Covered in the 400-level course

VOCABULARY

Co-function; identify; to verify an identity

ACTIVITIES

Logic diagrams and algebraic solutions

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 7

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapters 8 and 10

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 5 : Trigonometric Equations

ESSENTIAL QUESTIONS

1. Can the student distinguish between the graph of a sine/cosine function and that of a tangent function?
2. Can the student determine the inverse of a function?
3. Can the student algebraically solve a trig equation?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Geometry from an Algebraic Perspective; Trigonometry

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Graphs of trig equations
2. Inverse trig functions
3. General solution of a trig equation
4. Solution over a specified domain for a trig equation

Skills:

1. Use trig identities (for substitution) in solving trig equations
2. Use algebraic factoring, quadratic formula, and other algebraic techniques to solve trig equations
3. Determine if the inverse equation is a function
4. Solve an inverse trig function for its principal value
5. Distinguish between sin and Sin .

*****Covered in the 400-level course

VOCABULARY

Arcsine; complete graph; domain; general solution; interval notation; inverse relation; inverse trig function; principal value; principal value solution

ACTIVITIES

Graphing using calculator

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapters 6 and 7

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapters 8 and 10

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

***UNIT 6 : Parametric Equations**

ESSENTIAL QUESTIONS

1. Can the student use parametric equations to solve problems related to the motion of a projectile, its trajectory, and range?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Functions; Geometry from an Algebraic Perspective; Trigonometry

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Parametric equations of a line
2. Parametric equations for the path of a projectile
3. Parametric equations for the distance formula

Skills:

1. Given a problem situation, write the related parametric equations
2. Use the graphing calculator to graph parametric equations

*****Covered in the 400-level course

*VOCABULARY

Horizontal velocity; initial velocity; initial vertical velocity; magnitude; parametric equations; projectile; T-step

ACTIVITIES

Calculator programming and graphing of equations

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 8

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 12

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 7 : Relations; Functions; Inverse Equations; Systems; and Graphs

ESSENTIAL QUESTIONS

1. Can the student determine whether or not a relation is also a function?
2. Can the student determine whether one equation is the inverse of another?
3. Can the student graph a linear system of inequalities?
4. Can the student describe how a graph is related to its parent graph through one or more translations?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Discrete Mathematics; Geometry from a Synthetic Perspective; Geometry from an Algebraic Perspective; Conceptual Underpinnings of Calculus; Mathematical Structure

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Linear, quadratic, cubic, absolute value, irrational and greatest integer functions
2. Composition of functions
3. Linear programming
4. Matrices
5. Line symmetry and point symmetry
6. Families of graphs

Skills:

1. Use algebraic techniques to solve any function type listed above
2. Graph any function type listed above
3. State the domain and range of the functions listed above
4. Graph a linear system of inequalities and locate the optimum point (linear programming)
5. Use algebraic techniques to solve a system of polynomial equations
6. Use matrices to solve a linear system of three or more equations
7. Use matrices to answer questions in a real world problem situation
8. Given three noncollinear points, use matrices to determine the related quadratic equation

9. Use the graphing calculator to determine the maximum in an area or volume function for a given problem situation
10. Simplify irrational expressions

*****Covered in the 400-level course

VOCABULARY

Absolute value; complete algebraic graph; composition of functions; dependent variable; discriminant; domain; even function; families of graphs; function; independent variable; inverse equation; inverse function; interval notation; line symmetry; odd function; parent graph; perpendicular bisector; point symmetry; range; real roots; relation; root; solution; system of equations; translation of a graph; zero

ACTIVITIES

Plotting and graphing

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapters 1-3

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapters 3 and 4

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 8: Polynomial Functions and Rational Functions

ESSENTIAL QUESTIONS

1. Can the student read a graph to estimate the real zeros (roots) of a polynomial function?
2. Can the student determine the number of real zeros for a given polynomial function?
3. Can the student locate any point of discontinuity or line of discontinuity for a rational function?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Rational functions
2. Polynomial functions

Skills:

1. Given one root of a polynomial function, determine the other roots
2. Given the roots of a polynomial function, determine the equation using integral coefficients
3. Use the graphing calculator to clarify line and point discontinuity
4. Sketch an appropriate graph from the written description of a polynomial function
5. Sketch an appropriate graph of a polynomial function, given its equation
6. Sketch an appropriate graph of a rational function, given its equation
7. Factor a polynomial function, over the set of rationals
8. Given the sketch of a polynomial function, state a related equation
9. Distinguish between roots and factors
10. Distinguish between roots and real roots
11. Understand the relationship between real roots and x-intercepts

*****Covered in the 400-level course

VOCABULARY

Complete graph; complex roots; discriminant; factors; horizontal asymptote; integral coefficients; line of discontinuity; point of discontinuity; restricted domain; root; slant asymptote; solution; synthetic substitution (division); vertical asymptote; zero

ACTIVITIES

Plotting and graphing

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapters 3 and 4

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 2

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 9 : Exponential Functions and Logarithmic Functions

ESSENTIAL QUESTIONS

1. Can the student recognize that an exponential function is appropriate to solve a problem situation?
2. Can the student recall and extend the properties of exponents and logarithms?
3. Can the student solve an exponential equation?
4. Can the student solve a logarithmic equation?
5. Can the student interpret real world situations and solve?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Mathematical Structure

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Rational exponents and irrational exponents
2. Exponential expressions
3. Logarithmic expressions
4. Exponential functions
5. Logarithmic functions

Skills:

1. Use the properties of exponents and logarithms to simplify expressions
2. Use the properties of exponents and logarithms to evaluate expressions
3. Solve equations by using the properties of exponents and logarithms
4. Graph exponential functions
5. Graph logarithmic functions
6. Rewrite an exponential expression to contain no negative exponents
7. Rationalize the denominator of any expression
8. Read and interpret real world situations and solve using exponential functions

*****Covered in the 400-level course

VOCABULARY

Argument; base; common logarithm; compounded – continuously or over a specified interval; decreasing function; *e;* exponent; exponential decay; exponential growth; half-life; increasing function; *ln;* natural logarithms; power

ACTIVITIES

Evaluating real world situations; graphing

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 11

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 5

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 10 : Arithmetic and Geometric Sequences and Series

ESSENTIAL QUESTIONS

1. Can the student recognize arithmetic and geometric sequences?
2. Can the student write examples of arithmetic and geometric sequences?
3. Can the student determine whether a series is convergent or divergent?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Discrete Mathematics

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Arithmetic sequences and series
2. Geometric sequences and series
3. Infinite sequences and series

Skills:

1. Identify arithmetic, geometric and infinite series
2. Find sums of arithmetic, geometric, and infinite series (where possible)
3. Find the *n*th term of an arithmetic or geometric sequence
4. Find the limit of the terms of an infinite sequence

*****Covered in the 400-level course

VOCABULARY

Arithmetic mean; arithmetic sequence; arithmetic series; common difference; common ratio; convergent series; divergent series; geometric mean; geometric sequence; geometric series; infinite sequence; infinite series; limit; recursive

ACTIVITIES

Using calculator

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 12

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 13

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 11 : Conics

ESSENTIAL QUESTIONS

1. Can the student recognize an equation whose graph is either a circle, ellipse, parabola, or hyperbola?
2. Can the student use completing the square, an algebraic technique, to transform a conics equation from general to standard form?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Algebra; Functions; Geometry from an Algebraic Perspective

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Circle
2. Ellipse
3. Parabola
4. Hyperbola

Skills:

1. Identify the type of conic from a general equation
2. Rewrite an equation from general to standard form
3. Given an equation in standard form, identify the conic and sketch a complete, reasonable graph
4. Given the graph of a conic function, state a related equation
5. Identify the coordinates of all vertices and foci
6. Identify the equation of the directrix
7. Identify the equations of any asymptotes
8. Solve a system of equations for all points of intersection

*****Covered in the 400-level course

VOCABULARY

*Analytical geometry; circle; directrix; distance formula; *eccentricity; ellipse; focus (foci); general form of an equation; hyperbola; major axis; minor axis; parabola; slant asymptote; standard form of an equation; vertex (vertices)

ACTIVITIES

Drawing; graphing, plotting

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level: **Advanced Mathematical Concepts.** Merrill, 1999
Chapter 10

400-level: **Advanced Mathematics.** McDougal Littell Houghton Mifflin, 1994
Chapter 6

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 12 : Limits; Derivatives and Curve Sketching

ESSENTIAL QUESTIONS

1. Can the student determine the limit of a polynomial function?
2. Can the student determine the derivative of a function?
3. Can the student visually locate local maximum/minimum locations on a graph?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Functions; Conceptual Underpinnings of Calculus

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. First and second derivatives
2. Limits

Skills:

1. Define the limit of $f(x)$
2. Use derivative rules to determine $f'(x)$
3. Determine the slope of the tangent to the curve at any point
4. Use algebraic techniques (factoring, multiplying by the conjugate of a radical expression) to modify the quotient of two functions so that a limit can be determined
5. Evaluate a limit or determine that it does not exist
6. Determine the slope of the tangent to the curve at any point
7. Use derivatives to locate local maximum/minimum points
8. Use derivatives to locate any points of inflection

*****Covered in the 400-level course

VOCABULARY

Binomial conjugate; derivative; point of inflection; point-slope form of a linear equation; relative (local) maximum; relative (local) minimum; secant line; slope; tangent to a curve

ACTIVITIES

Graphing; real world problems

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

300-level **Advanced Mathematical Concepts.** Merrill, 1999
Chapters 3 and 17

400-level: **Advanced Mathematics.**. McDougal Littell Houghton Mifflin, 1994
Chapters 19 and 20

ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

UNIT 12 : Algebraic Methods of Data Analysis

ESSENTIAL QUESTIONS

1. Can the student interpret data tables?
2. Can the student determine the process to create functions and equations from data?
3. Can the student understand different methods to use discrete mathematical models?
4. Can the student write the appropriate equation based on the data?

NCTM STANDARDS

Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; Mathematical Connections; Functions; Conceptual Underpinnings of Calculus

CONTENT KNOWLEDGE OBJECTIVES

Content:

1. Recursively defined functions
2. Modeling growth
3. Best fit line
4. Least squares line
5. Correlation and accuracy

Skills:

1. Using the graphing calculator to plot data
2. Using the calculator to determine line of best fit
3. Interpreting word problems and translating into data
4. Interpreting data and translating into equations

VOCABULARY

Scientific notaton, sequences, depreciation, median, mean, extrapolation, linear, least-squares, correlation, assessing, projectile, inverse

ACTIVITIES

Graphing; real world problems, data tables

PERFORMANCE ASSESSMENT

Homework, quizzes, tests, class participation.

CAREER AWARENESS (where appropriate)

Engineering, architecture, teaching, construction, fine arts, athletics, astronomy, business

CORE TEXT FOR STUDENTS

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ADDITIONAL TEXTS/RESOURCES FOR USE BY STUDENTS

See Section IV, Learning Resources

MATERIALS AND SUPPLIES

See Section IV, Learning Resources

SECTION III - COURSE FORMULAS and STANDARDS

Algebraic Formulas

Polynomial Function $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$

Exponential Functions

$y = ab^x$ or $f(t) = a_0b^t$

Compounded Continuously $P = P_0 \cdot e^{rt}$

Compounded over Intervals $P = P_0\left(1 + \frac{r}{n}\right)^{nt}$

Half - life $P = P_0(0.5)^t$

Logarithmic Function $f(x) = \log_b x$

*n*th Term of Arithmetic Sequence $a_n = a_1 + (n-1)d$

Sum of Arithmetic Series $S_n = \frac{n}{2}(a_1 + a_n)$

*n*th Term of Geometric Sequence $a_n = a_1 \cdot r^{n-1}$

Sum of Geometric Series $S_n = \frac{a_1(1-r^n)}{1-r}$, $r \neq 1$

Sum of Infinite Series $S = \frac{a_1}{1-r}$

Circle $(x-h)^2+(y-k)^2=r^2$

Ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$

Parabola $y-k=\frac{1}{4p}(x-h)^2$ or $x-h=\frac{1}{4p}(y-k)^2$

Hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Derivative Definition $f'(x)=\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$

Derivative Rules

For $f(x)=c,\ f'(x)=0.$

For $f(x)=x^n,\ f'(x)=n\cdot x^{n-1},$ where n is a rational number.

For $f(x)=c\cdot x^n,\ f'(x)=cn\cdot x^{n-1},$ where c and n are rational numbers.

For $f(x)=g(x)+h(x),$ then $f'(x)=g'(x)+h'(x).$

Trigonometric Formulas

Conversions π radians = 180^0

Arc Length $s = r \cdot \theta$

Sector Area $A = \frac{1}{2} r^2 \theta$ or $K = \frac{1}{2} rs$

Trig Functions $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

Soh Cah Toa $\sin\theta = \frac{\text{oppposite leg}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$c^2 = a^2 + b^2 - 2ab \cdot \cos C$

Triangle Area $K = \frac{1}{2} ab \cdot \sin C$

$K = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

Sinusoidal Functions $y - k = A\cos(B(x-h))$ or $y - k = A\sin(B(x-h))$

Pythagorean Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

Reciprocal Identities

$$\sec A = \frac{1}{\cos A} \qquad \csc A = \frac{1}{\sin A} \qquad \cot A = \frac{1}{\tan A}$$

* Co - Function Identities

$$\sin\theta = \cos(\pi - \theta) \qquad \tan\theta = \cot(\pi - \theta) \qquad \sec\theta = \csc(\pi - \theta)$$

* Sum – Difference Identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mu \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mu \tan A\tan B}$$

Double Angle

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

* Parametric Equations (Line)

$$x = x_1 + t \cdot A_1 \quad \text{and} \quad y = y_1 + t \cdot A_2$$

* Parametric Equations (Projectile Path)

$$x = t\left|\vec{v}\right|\cos\theta \quad \text{and} \quad y = t\left|\vec{v}\right|\sin\theta + \frac{1}{2}gt^2$$

NCTM STANDARDS

1. Mathematics as Problem Solving

The mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that students can:

- use, with increasing confidence, problem solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems form situations within and outside mathematics;
- apply the process of mathematical modeling to real world situations.

2. Mathematics as Communication

The mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that students can:

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power and elegance of mathematical notation and its role in the development of mathematical ideas.

3. Mathematics as Reasoning

The mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that students can:

- make and test conjectures;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments.

4. Mathematical Connections

The mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that students can:

- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;
- use and value the connections among mathematical topics;
- use and value the connections between mathematics and other disciplines.

5. Algebra

The mathematics curriculum should include the continued study of algebraic concepts and methods so that students can:

- represent situations that involve variable quantities with expressions, equations, inequalities and matrices;
- use tables and graphs as tools to interpret expressions, equations and inequalities;
- operate on expressions and matrices and solve equations and inequalities;
- appreciate the power of mathematical abstraction and symbolism;
- demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

6. Functions

The mathematics curriculum should include the continued study of functions so that the student can:

- model real world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal rules, equations and graphs;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions;
- understand operations on, and the general properties and behavior of, classes of functions.

7. Geometry from a Synthetic Perspective

The mathematics curriculum should include the continued study of the geometry of two and three dimensions so that students can:

- interpret and draw three dimensional objects;
- represent problem situations with geometric models and apply properties of figures.

8. Geometry from an Algebraic Perspective

The mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that students can:

- translate between synthetic and coordinate representations;
- deduce properties of figures using transformations and using coordinates;
- apply transformations, coordinates and vectors in problem solving.

9. Trigonometry

The mathematics curriculum should include the study of trigonometry so that students can:

- apply trigonometry to problem situations involving triangles;
- explore periodic real world phenomena using the sine and cosine functions;
- understand the connection between trigonometric and circular functions;

- use circular functions to model periodic real world phenomena;
- apply general graphing techniques to trigonometric functions;
- solve trigonometric equations and verify trigonometric identities.

10. Discrete Mathematics
The mathematics curriculum should include topics from discrete mathematics so that students can:

- represent problem situations using discrete structures such as finite graphs, matrices, sequences and recurrence relations.
- develop and analyze algorithms;
- represent and solve problems using linear programming and difference equations.

11. Conceptual Underpinnings of the Calculus
The mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so the students can:

- determine maximum and minimum points of a graph and interpret result in problem situations;
- investigate limiting processes;
- understand the conceptual foundations of limit, rate of change and slopes of a tangent line and their applications in other disciplines;
- analyze the graphs of polynomial, rational, radical and transcendental functions.

12. Mathematical Structure
The mathematics curriculum should include the study of mathematical structure so that students can:

- understand the logic of algebraic procedures;
- appreciate that seemingly different mathematical systems may be essentially the same;
- demonstrate an understanding of the nature and purpose of axiomatic systems.

Connecticut Content Standards

1. Number Sense Students will use numbers to count, measure, compare, order, scale, locate and label, and use a variety of numerical representations to present, interpret, communicate and connect various kinds of numerical information.

2. Operations Students will add, subtract, multiply and divide with whole numbers, fractions, decimals and integers, and develop strategies for selecting the appropriate computational and operational methods for solving problems.

3. Estimation and Approximation Students will make estimates and approximations, and judge the reasonableness of results.

4. Ratios, Proportions and Percents Students will use ratios, proportions and percents to represent relationships between quantities and measures and solve problems involving ratios, proportions and percents.

5. Measurement Students will make and use measurements in both customary and metric units to approximate, measure and compute length, area, volume, mass, temperature, angle and time.

6. Spatial Relationships and Geometry Students will analyze and use spatial relationships and basic concepts of geometry to construct, draw, describe and compare geometric models and their transformations, and use geometric relationships and patterns to solve problems.

7. Probability and Statistics Students will use basic concepts of probability and statistics to collect, organize, display and analyze data, simulate events and test hypotheses.

8. Patterns Students will discover, analyze, describe, extend and create patterns, and use patterns to describe mathematical and other real-world phenomena.

9. Algebra and Functions Students will use algebraic skills and concepts, including functions, to describe real-world phenomena symbolically and graphically, and to model quantitative change.

10. Discrete Mathematics Students will use the concepts and processes of discrete mathematics to analyze and model a variety of real-world situations that involve recurring relationships, sequences, networks, combinations and permutations.

SECTION IV -- LEARNING RESOURCES

SUPPLEMENTAL RESOURCES (Most located in math department)

Text books

- Precalculus with Trigonometry, Addison Wesley, 1987
- Precalculus, Stewart, et. al., Brooks/Cole Publishing, 2002
- Advanced Algebra through Data Exploration, Key Curriculum Press, 1998

Websites

- www.nctm.org
- www.learner.org/exhibits/dailymath
- www.math.temple.edu/~paulos
- www.mathforum.org
- www.maa.org
- http://www.sosmath.com/algebra/algebra.html
- http://www.algebrahelp.com
- www.mathematicallycorrect.com
- www.personal.cfw.com/~clayford
- www.math.com
- www.math.uah.edu/psol
- www.nilesonline.com/stats
- www.mathmistakes.com
- www.innumeracy.com
- www.techlar.com/fractals
- www.superstringtheory.com

Other Resources

- TI-83 or TI-83 Plus graphing calculator
- Numerous Teacher worksheets
- PowerPoint presentations
- TTL/C-5 Computer Lab
- Microsoft Excel
- World Almanac
- Media center
- Winplot
- LiveMath
- Access
- Internet