ARE YOU READY FOR CALCULUS?

Congratulations! You made it to AB/BC Calculus!

Instructions
1. Please complete this packet which will be due the first day of school on August 29th. This packet will help you review for the test that you will have on the second day of school.
2. Do your work on a separate sheet of paper clearly and neatly.

SECTION 1.1 LEARNING OBJECTIVES
1. Be able to calculate the slope of a line
2. Be able to determine the equation of a line using slope-intercept form, point-slope form and the general form.
3. Know the relationship of slopes for parallel and perpendicular lines.
4. Be able to create a linear regression equation from data and use it to make predictions.

SECTION 1.2 LEARNING OBJECTIVES
1. Know the definition of a function.
2. Be able to determine the domain and range of a function.
3. Be able to express answers in interval notation.
4. Be able to determine symmetry properties of a function.
5. Be able to identify odd and even functions.
6. Be able to graph piecewise functions.
7. Be able to compose functions.

SECTION 1.3 LEARNING OBJECTIVES
1. Know the rules for operating with exponents.
2. Be able to create an exponential growth/decay equation.
3. Be able to solve exponential equations.

SECTION 1.4 LEARNING OBJECTIVES
1. Know how to graph a parametric equation, indicate initial and terminal points and direction in which it is traced (orientation).
2. Be able to convert a parametric equation into rectangular (Cartesian) form.
3. Be able to parametrize an equation.

SECTION 1.5 LEARNING OBJECTIVES
1. Be able to identify one-to-one functions.
2. Be able to determine the algebraic and graphical representation of a function and its inverse.
3. Be able to apply the properties of logarithms.

SECTION 1.6 LEARNING OBJECTIVES
1. Be able to convert between radians and degrees.
2. Be able to find arc length.
3. Be able to generate the graphs of the trigonometric functions.
4. Be able to identify periodicity and the even-odd properties of the trigonometric functions.
5. Be able to use the inverse trigonometric functions to solve problems.
6. Be able to identify period, amplitude, domain, range and various translations of the trigonometric functions.

Forming Functions LEARNING OBJECTIVES from Verbal Descriptions
1. To form a function of one variable from a verbal description
2. Determine the minimum or maximum value of the function

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

6. $f(2) =$ ____________ 7. $g(-3) =$ ______________ 8. $f(t+1) =$ __________

9. $f\left[g(-2)\right] =$__________ 10. $g\left[f(m+2)\right] =$___________ 11. $\dfrac{f(x+h) - f(x)}{h} =$______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$___________ 13. $f\left(\dfrac{2\pi}{3}\right) =$_______________

Let $f(x) = x^2,\ g(x) = 2x + 5,$ *and* $h(x) = x^2 - 1$**. Find each.**

14. $h\left[f(-2)\right] =$ _______ 15. $f\left[g(x-1)\right] =$ _______ 16. $g\left[h(x^3)\right] =$ _______

Optional: Do this section *if* you need the review.

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$x-$int. (*Let* $y=0$)	$y-$int. (*Let* $x=0$)
$0 = x^2 - 2x - 3$	$y = 0^2 - 2(0) - 3$
$0 = (x-3)(x+1)$	$y = -3$
$x = -1$ *or* $x = 3$	$y-$intercept $(0,-3)$
$x-$intercepts $(-1,0)$ *and* $(3,0)$	

Find the x and y intercepts for each.

19. $y = 2x - 5$

20. $y = x^2 + x - 2$

21. $y = x\sqrt{16 - x^2}$

22. $y^2 = x^3 - 4x$

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9$$
$$= x-9+9$$
$$= x$$

$$g(f(x)) = \frac{(4x+9)-9}{4}$$
$$= \frac{4x+9-9}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Equation of a line

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line passing through the point having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

Reference Triangles

62. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

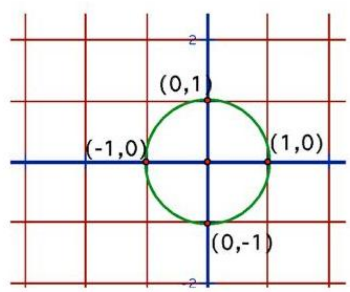
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

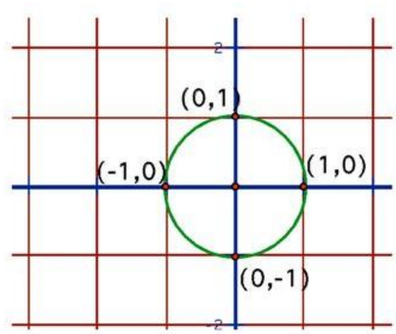
Example: $\sin 90^{\circ} = 1$ $\cos\frac{\pi}{2} = 0$



63. $a.)\ \sin 180^{\circ}$ $b.)\ \cos 270^{\circ}$

$c.)\ \sin(-90^{\circ})$ $d.)\ \sin\pi$

$e.)\ \cos 360^{\circ}$ $f.)\ \cos(-\pi)$



Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

68. $\sin x = -\frac{1}{2}$

69. $2\cos x = \sqrt{3}$

70. $\cos 2x = \frac{1}{\sqrt{2}}$

71. $\sin^2 x = \frac{1}{2}$

72. $\sin 2x = -\frac{\sqrt{3}}{2}$

73. $2\cos^2 x - 1 - \cos x = 0$

74. $4\cos^2 x - 3 = 0$

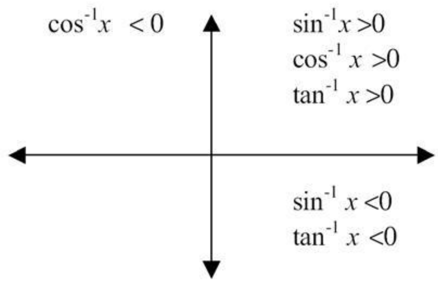
75. $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad \text{OR} \qquad \sin^{-1}(x)$$

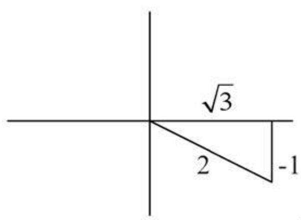
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = – $\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: y = – $\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

76. $y = \arcsin\frac{-\sqrt{3}}{2}$

77. $y = \arccos(-1)$

78. $y = \arctan(-1)$

Digital Lesson

Graphs of Trigonometric Functions

Properties of Sine and Cosine Functions

The graphs of $y = \sin x$ and $y = \cos x$ have similar properties:

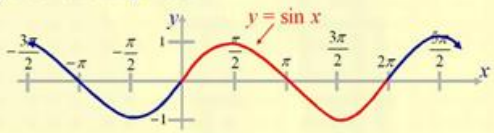
1. The domain is the set of real numbers.
2. The range is the set of y values such that $-1 \le y \le 1$.
3. The maximum value is 1 and the minimum value is -1.
4. The graph is a smooth curve.
5. Each function cycles through all the values of the range over an x-interval of 2π.
6. The cycle repeats itself indefinitely in both directions of the x-axis.

Graph of the Sine Function

To sketch the graph of $y = \sin x$ first locate the key points. These are the maximum points, the minimum points, and the intercepts.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0

Then, connect the points on the graph with a smooth curve that extends in both directions beyond the five points. A single cycle is called a **period**.

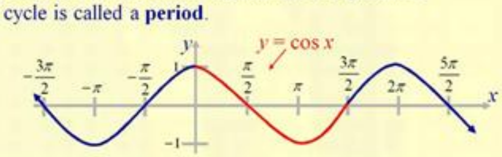


Graph of the Cosine Function

To sketch the graph of $y = \cos x$ first locate the key points. These are the maximum points, the minimum points, and the intercepts.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1

Then, connect the points on the graph with a smooth curve that extends in both directions beyond the five points. A single cycle is called a **period**.



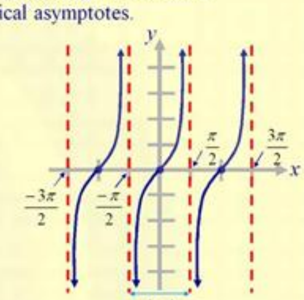
Graph of the Tangent Function

To graph $y = \tan x$, use the identity $\tan x = \frac{\sin x}{\cos x}$.

At values of x for which $\cos x = 0$, the tangent function is undefined and its graph has vertical asymptotes.

Properties of $y = \tan x$

1. domain : all real x
   $x \ne k\pi + \frac{\pi}{2} \,(k \in Z)$
2. range: $(-\infty, +\infty)$
3. period: π
4. vertical asymptotes:
   $x = k\pi + \frac{\pi}{2} \,(k \in Z)$



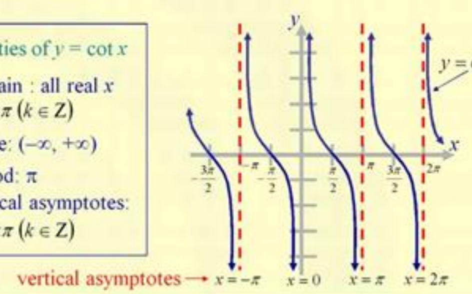
Graph of the Cotangent Function

To graph $y = \cot x$, use the identity $\cot x = \frac{\cos x}{\sin x}$.

At values of x for which $\sin x = 0$, the cotangent function is undefined and its graph has vertical asymptotes.

Properties of $y = \cot x$

1. domain : all real x
   $x \ne k\pi \,(k \in Z)$
2. range: $(-\infty, +\infty)$
3. period: π
4. vertical asymptotes:
   $x = k\pi \,(k \in Z)$



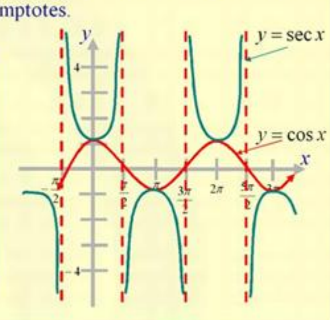
Graph of the Secant Function

The graph $y = \sec x$, use the identity $\sec x = \frac{1}{\cos x}$.

At values of x for which $\cos x = 0$, the secant function is undefined and its graph has vertical asymptotes.

Properties of $y = \sec x$

1. domain : all real x
   $x \ne k\pi + \frac{\pi}{2} \,(k \in Z)$
2. range: $(-\infty, -1] \cup [1, +\infty)$
3. period: π
4. vertical asymptotes:
   $x = k\pi + \frac{\pi}{2} \,(k \in Z)$



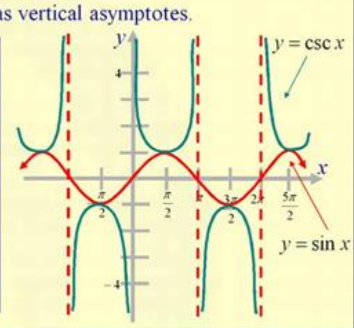
Graph of the Cosecant Function

To graph $y = \csc x$, use the identity $\csc x = \frac{1}{\sin x}$.

At values of x for which $\sin x = 0$, the cosecant function is undefined and its graph has vertical asymptotes.

Properties of $y = \csc x$

1. domain : all real x
   $x \ne k\pi \,(k \in Z)$
2. range: $(-\infty, -1] \cup [1, +\infty)$
3. period: π
4. vertical asymptotes:
   $x = k\pi \,(k \in Z)$
   where sine is zero.



DEFINITION 2.2. A function "has a limit." *We say that a function f(x) "has a limit" L as x approaches l, if for every sequence of values of x that approach l as a limit -- whether from the left or from the right -- the corresponding values of f(x) approach L as a limit.*

If that is the case, then we write:

$$\lim_{x \to l} f(x) = L$$

"The limit of *f*(*x*) as *x* approaches *l* *is* *L*."

In other words, for the limit of *f*(*x*) to *exist* as *x* approaches *l*, the left-hand and right-hand limits must be equal.

$$\lim_{x \to l} f(x) = L$$

if and only if

$$\lim_{x \to l-} f(x) = L \quad \text{and} \quad \lim_{x \to l+} f(x) = L.$$

We will see that this definition of the limit of a function *existing* as *x* approaches *l*, becomes the definition of the function being *continuous* at that value -- if *L* is the *value* of the function there; that is, if *L* = *f*(l).

Limits

Finding limits numerically.

Complete the table and use the result to estimate the limit.

87. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

88. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

89. $\lim_{x \to 0} \cos x$

90. $\lim_{x \to 5} \frac{2}{x-5}$

91. $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

92. $\lim_{x \to 2} (4x^2+3)$

93. $\lim_{x \to 1} \frac{x^2+x+2}{x+1}$

94. $\lim_{x \to 0} \sqrt{x^2+4}$

95. $\lim_{x \to \pi} \cos x$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

105. $f(x)=\frac{1}{x^2}$

106. $f(x)=\frac{x^2}{x^2-4}$

107. $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

108. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

109. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

110. $f(x)=\frac{4x^5}{x^2-7}$

Limits to Infinity

A rational function does not have a limit if it goes to $\pm\infty$, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction.

Determine each limit if it exists. If the limit approaches ∞ *or* $-\infty$, please state which one the limit approaches.

111. $\lim\limits_{x\to -1^+}\frac{1}{x+1}=$

112. $\lim\limits_{x\to 1^+}\frac{2+x}{1-x}=$

113. $\lim\limits_{x\to 0}\frac{2}{\sin x}=$

REQUIRED collection of problems: TO DO & HAND IN

1) Simplify:

$a)\dfrac{x^3-9x}{x^2-7x+12}$ $b)\dfrac{x^2-2x-8}{x^3+x^2-2x}$ $c)\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ $d)\dfrac{9-x^{-2}}{3+x^{-1}}$

2) Rationalize the denominator:

$a)\dfrac{2}{\sqrt{3}+\sqrt{2}}$ $b)\dfrac{4}{1-\sqrt{5}}$ $c)\dfrac{1}{1+\sqrt{3}-\sqrt{5}}$

3) Write each of the following expressions in the form
ca^pb^q**, where c,p and q are numbers:**

$a)\dfrac{(2a^2)^3}{b}$ $b)\sqrt{9ab^3}$ $c)\dfrac{a\left(2/b\right)}{3/a}$ $d)\dfrac{ab-a}{b^2-b}$ $e)\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$ $f)\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

4) Solve for x (do not use a calculator):

$a)5^{(x+1)}=25$ $b)\dfrac{1}{3}=3^{2x+2}$ $c)\log_2 x=3$ $d)\log_3 x^2=2\log_3 4-4\log_3 5$

5) Simplify:

$a)\log_2 5+\log_2(x^2-1)-\log_2(x-1)$ $b)2\log_4 9-\log_2 3$ $c)3^{2\log_3 5}$

6) Simplify:

$a)\log_{10}\left(10^{\frac{1}{2}}\right)$ $b)\log_{10}\left(\dfrac{1}{10^x}\right)$ $c)2\log_{10}\sqrt{x}+3\log_{10}x^{\frac{1}{3}}$

7) Solve the following equations for the indicated variables:

$a)\ \dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1,\ for\quad a$ $b)\ V=2(ab+bc+ca),\ for\quad a$

$c)\ A=2\pi r^2+2\pi rh,\ for\quad positive\quad r$ $d)\ 1+xz+y+4yz=0\ \ for\quad z$

$e)\ 2x-2yd=y+xd,\ for\quad d$ $f)\ \dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0,\ for\quad x$

8) For the following equations, complete the square and reduce to one of the standard forms

$y-b=A(x-a)^2 \; or \quad x-a=A(y-b)^2$

$a)\, y=x^2+4x+3 \qquad b)\; 3x^2+3x+2y=0 \qquad c)\; 9y^2-6y-9-x=0$

9) Factor completely:

$a)\; x^6-16x^4 \qquad b)\; 4x^3-8x^2-25x+50 \qquad c)\; 8x^3+27 \qquad d)\, x^4-1$

10) Find all real solutions to:

$a)x^6-16x^4=0 \qquad b)4x^3-8x^2-25x+50=0 \qquad c)8x^3+27=0 \qquad d)x^4-1=0$

11) Solve for x:

$a)3\sin^2 x=\cos^2 x, \qquad 0\le x<2\pi$

$b)\cos^2 x-\sin^2 x=\sin x, \qquad -\pi<x\le\pi$

$c)\tan x+\sec x=2\cos x, \qquad -\infty<x<\infty$

12) Without using a calculator, evaluate the following:

$a)\cos 210^\circ \qquad b)\sin\frac{5\pi}{4} \qquad c)Tan^{-1}(-1) \qquad d)Sin^{-1}(-1)$

$e)\cos\frac{9\pi}{4} \qquad f)Sin^{-1}\frac{\sqrt{3}}{2} \qquad g)\tan\frac{7\pi}{6} \qquad h)\; Cos^{-1}(-1)$

13) Given the graph of sin x, sketch the graphs of:

$a)\sin(x-\frac{\pi}{4}) \qquad b)\sin\left(\frac{x}{2}\right) \qquad c)2\sin x \qquad d)\cos x \qquad e)\frac{1}{\sin x}$

14) Solve the equations:

$a)4x^2+12x+3=0 \qquad b)2x+1=\frac{5}{x+2} \qquad c)\frac{x+1}{x}-\frac{x}{x+1}=0$

15) Find the remainders on division of:

$a)x^5-4x^4+x^3-7x+1 \quad by \quad x+2$

$b)x^5-x^4+x^3+2x^2-x+4 \quad by \quad x^3+1$

16) **a) Find all other solutions is one solution is x=2 for the following equation:**

$$12x^3 - 23x^2 - 3x + 2 = 0$$

b) Solve for x if all the solutions are rational and between +/- 1.

$$12x^3 + 8x^2 - x - 1 = 0$$

17) Solve:

$a)\ x^2 + 2x - 3 \le 0$ $\qquad b)\ \dfrac{2x-1}{3x-2} \le 1$ $\qquad c)\ |5x-2| = 8$

18) For the function $f(x) = \dfrac{-x^2}{\sqrt{20-x^2}} + \sqrt{20-x^2}$ **find the values of x which make:**

a) ***f(x)*** **= 0**
b) ***f(x)*** **> 0**

19) Determine the equations of the following lines:

a) the line through (-1, 3) and (2, -4).
b) the line through (-1, 2) and perpendicular to the line 2x - 3y + 5 = 0.
c) the line through (2,3) and the midpoint of the segment from (-1,4) to (3,2).

20) Find the point of intersections:

a) $\begin{matrix} 3x - y - 7 = 0 \\ x + 5y + 3 = 0 \end{matrix}$ $\qquad$ **b)** $\begin{matrix} 4x^2 + 4y^2 = 25 \\ 2x + y^2 = 1 \end{matrix}$

21) Find the equations of the following circle:

The circle with center at (1, 2) that passes through the point (-2, -1).

22) For the following circle,

$$x^2 + y^2 + 6x - 4y + 3 = 0$$

a) find the center and radius
b) the equation of the tangent at (-2, 5).

23) Find the domain of the function

$$f(x) = \frac{3x+1}{\sqrt{x^2 + x - 2}}$$

24) Find the domain and range of the following functions:

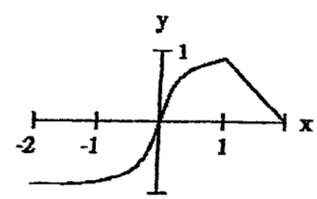
$$a)\ f(x)=\sqrt{4-x^2} \qquad b)\ g(x)=\frac{5x-3}{2x+1}$$

25) For each of the following functions find:

a) $f(x+h)$ b) $f(x+h)-f(x)$ c) $\frac{f(x+h)-f(x)}{h}$

1. $f(x)=2x+3$
2. $f(x)=\frac{1}{x+1}$
3. $f(x)=x^2$

26) The graph of the function y = f(x) is given as follows:



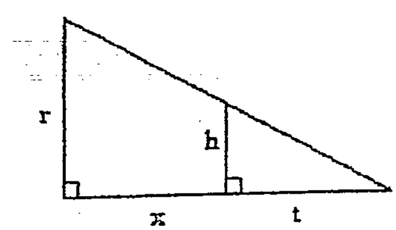
Determine the graphs of the functions:

$$a)\ f(x+1) \quad b)\ f(-x) \quad c)\ |f(x)| \quad d)\ f(|x|)$$

27) Find the inverse of the functions:

$$a)\ f(x)=2x+3 \quad b)\ f(x)=\frac{x+2}{5x-1} \quad c)\ f(x)=x^2+2x-1, x>0$$

28) Express x in terms of the other variables in the picture:



29) Let $f(x)=\frac{|x|}{x}$.

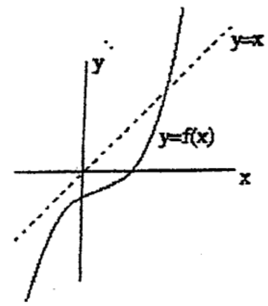
Show that $f(x)=\begin{cases}1 & x>0\\ -1 & x<0\end{cases}$

Find the domain and range of f(x).

30) a) The graph of a quadratic function (a parabola) has x-intercepts -1 and 3 and a range consisting of all values less than or equal to 4. Determine an expression for the function.

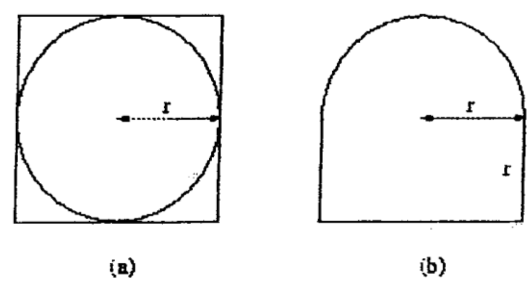
b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3.$

31) A function f(x) has the graph to the right.



Sketch the graph of the inverse $f^{-1}(x).$

32) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



b) Find a formula for the perimeter of a window of the shape in the picture (b) above.

c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

d) Two cars start moving from the same point. One travels south at 100 km/hour, the other west at 50 km/hour. How far apart are they two hours later?

e) A kite is 100 m above the ground. If there are 200 m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$ $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0